B.SC. THIRD SEMESTER (HONOURS) EXAMINATION 2021

Subject: Mathematics **Course ID: 32115**

Course code: SH/MTH/305/SEC-1 **Course Title: Logic and Sets**

Time: 2 Hours **Full Marks: 40**

<u>The figures in the margin indicate full marks</u>

Notations and symbols have their usual meaning

1. Answer any five questions: $2 \times 5 = 10$

   (a) Show that $R = \{(x, y): (x - y) \text{ is divisible by } 7\} \subset \mathbb{Z} \times \mathbb{Z}$ is an equivalence relation on $\mathbb{Z}$.

   (b) Let $A = \{x \in \mathbb{Z}: 0 \leq x \leq 10\}$ and $B = \{x \in \mathbb{Z}: x \leq 15\}$. Find $A - B$.

   (c) Determine the power set of $A = \{1, 2, 3, 4\}$.

   (d) Find all the equivalence relations on the set $S = \{a, b, c\}$.

   (e) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then prove that $B = C$.

   (f) Let A, B be two subsets of an universal set. Prove that $A = B$ if and only if $A \Delta B = \emptyset$.

   (g) Construct the truth table for the statement formula $(\sim p \vee \sim q)$.

   (h) Find the negation of the following quantified predicates:

   $$(\exists\, x, y \in D)(x + y = 3).$$

2. Answer any four question: $5 \times 4 = 20$

   (a) (i) For any three non-empty sets A, B, C prove that

   $$A \times (B \cap C) = (A \times B) \cap (A \cap C).$$

   (ii) Show that the sets $A = \{2, 1\}$ and $B = \{x \in \mathbb{R}: x^2 - 3x + 2 = 0\}$ are equal. 3+2

   (b) If $A = \{5, 6, 7, 8, 9\}$, $B = \{2,4,6,8,10,12\}$ and $C = \{3,6,9,12\}$, then find

   $$A \cap (B \cap C), A \cup (B \cup C), A \cup (B \cap C), A \cap (B \cup C), A - (B \cup C).$$

   $1 + 1 + 1 + 1 + 1 = 5$

   (c) (i) Prove that intersection of two equivalence relations on a set A is an equivalence relation on A.

   (ii) Is union of two equivalence relations an equivalence relation? Justify your answer. $3 + 2 = 5$

   (d) It is known that in an university, 60% of professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 40% play bridge and jog and 30% play tennis and jog. If someone claimed that 20% professors jog and play tennis and bridge, would you believe his claim? Justify.

(e) (i) Show that the propositions $\sim(a \vee b)$ and $\sim a \wedge \sim b$ are logically equivalent.

(ii) Construct the truth table for the statement form: $(a \vee \sim b) \wedge c$.

3. Answer any one question: $10 \times 1 = 10$

(a) (i) A relation ρ is defined on $\mathbb{Z}$ by "$x\rho y$ iff $2x + 3y$ is divisible by 5". Prove that ρ is an equivalence relation on $\mathbb{Z}$.

(ii) If $A\Delta B = A\Delta C$, then prove that $B = C$

(iii) Show that $(A - B)$ and $(A \cap B)$ are disjoint sets. 5+3+2

(b) (i) Let p, q, r be three statements. Show that $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ (using truth table).

(ii) A relation ρ is defined on the set $\mathbb{Z}$ by "$x\rho y$ if and only if $x + y$ is odd" for $x, y \in \mathbb{Z}$. Examine whether ρ is reflexive, symmetric and transitive.

(iii) Let ρ be a binary relation on a set A. Then prove that ρ is transitive if and only if $\rho o \rho \subset \rho$. 4+3+3

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